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※ Your answer must be provided with descriptions how to get the answer.

1. (3 points) Calculate the iterated integral

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\ dydx.$$

2. (3 points) Find the volume of the solid that lies above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

3. (a) (2 points) Show that the vector field $\boldsymbol{F}(x,\ y,\ z) = < ye^{xy},\ xe^{xy} + e^z, ye^z >$ is conservative.

(b) (3 points) Find a function f such that $\boldsymbol{F} = \nabla f$.

(c) (1 point) Calculate the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where C is the line segment from $(0,0,0)$ to $(1,1,0)$.

4. (4 points) Evaluate $\oint_C -x^2y\,dx + xy^2\,dy$ where C is the circle $x^2+(y-1)^2=1$.

5. (4 points) Let S be a parametric surface given by the equation

$$x=u\cos\theta,\ y=u\sin\theta,\ z=1-u\cos\theta-u\sin\theta,$$

$0 \le u \le 2,\ 0 \le \theta \le 2\pi$. Find the area of the surface.

6. (4 points) Evaluate the integral.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z\ dz\,dy\,dx$$

$$+\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_{\csc\phi}^{2} \rho^3\cos\phi\sin\phi\ d\rho\,d\phi\,d\theta$$

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7. (4 points) Find the flux of the vector field $\boldsymbol{F}(x,y,z) = -yz^2e^{xy}\boldsymbol{i} + xz^2e^{xy}\boldsymbol{j} + z\boldsymbol{k}$ across S, where S is the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$, and has downward orientation.

8. (4 points) Evaluate the integral $\iint_R (x+y)\, e^{x^2-y^2}\, dA$ by making an appropriate change of variables where R is the rectangle enclosed by the lines $x-y=0$, $x-y=1$, $x+y=0$, and $x+y=2$.

9. (4 points) Using the Divergence Theorem, evaluate

$$\iint_S < xz^2, \frac{1}{3}y^3 + \tan z, x^2 z + y^2 > \cdot \, d\mathbf{S},$$

where S is the top half of the sphere $x^2 + y^2 + z^2 = 1$.

10. (4 points) Evaluate the integral $\iint_S \mathrm{curl}\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = ye^{yz}\cos z\,\mathbf{i} + x e^{xz}\sin z\,\mathbf{j} + e^{xy}\,\mathbf{k}$ and S is the hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$, oriented in the direction of the positive z-axis.