CALCULUS II
2022 Fall Final Exam

Dept. or School		Year		proctor	
Student ID		Name			

※ Your answer must be provided with descriptions how to get the answer.

1. (3 points) Find the volume of a solid that is bounded by $x=4-\frac{z^2}{4}-y^2$ and $x=-1+z^2+\frac{y^2}{4}$.

2. (3 points) Evaluate $\iiint_E z\,dV$ where E is enclosed by the paraboloid $z=x^2+y^2$ and the plane $z=4$.

3. (3 points) Evaluate the integral by changing to spherical coordinates.

$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2+\sqrt{4-x^2-y^2}}\frac{z}{x^2+y^2+z^2}\,dz\,dy\,dx$$

4. Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ of

$$\mathbf{F}(x, y) = < \sin x \cos y,\ e^{xy} > = \sin x \cos y\,\mathbf{i} + e^{xy}\mathbf{j}$$

along

(a) (2 points) the horizontal line $C_1 : 0 \le x \le \pi,\ y = \frac{\pi}{3}$

(b) (2 points) the vertical line $C_2 : 0 \le y \le 1,\ x = 2$

5. (3 points) Evaluate $\iint_D \frac{\cos\sqrt{9x^2+4y^2}}{\sqrt{9x^2+4y^2}}\,dA$ where D is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.

6. Let $\mathbf{F}(x,y,z) = (3x^2yz - 3y)\mathbf{i} + (x^3z - 3x)\mathbf{j} + (x^3y + 2z)\mathbf{k}$.

(a) (2 points) Find a function f such that $\nabla f = \mathbf{F}$

(b) (2 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the straight line with initial point $(0,0,2)$ and terminal point $(0,3,0)$.

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7. Let C be the astroid $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}$, $(0 \le t \le 2\pi)$.

(a) (3 points) Find the area of the region enclosed by the curve C.

(b) (2 points) Find the work done by the force field $\mathbf{F}(x,y) = (e^x + \tan^{-1} y)\mathbf{i} + (3x + \dfrac{x}{1+y^2})\mathbf{j}$ in moving a particle once counterclockwise around the curve C.

8. (3 points) Let $\mathbf{F} = (2x - y + az)\mathbf{i} + (bx + y - 3z)\mathbf{j} + (x - cy + 5z)\mathbf{k}$. Find a, b, and c so that $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

In this case, evaluate the divergence of $\mathbf{F}$.

9. (3 points) Find the area of the surface

$$S:\ \rho = 1,\ 0 \le \theta \le \pi,\ 0 \le \cos\phi \le \sin\theta\ .$$

10. (3 points) Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, where $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and S is the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

11. (3 points) Using the Stokes' Theorem, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = (2yz + e^{x^2})\mathbf{i} + \tan^{-1} y\,\mathbf{j} + xy\mathbf{k}$ and C is the boundary of the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant where C is oriented counterclockwise when viewed from above.

12. (3 points) Using Divergence Theorem, calculate the flux of $\mathbf{F}$ across S where $\mathbf{F}(x, y, z) = e^y \tan z\mathbf{i} + x^2 y\mathbf{j} + e^x \cos y\mathbf{k}$, S is the surface of the solid that lies above the xy-plane and below the surface $z = 2 - x - y^3$, $-1 \le x \le 1$, $-1 \le y \le 1$.