CALCULUS II **2021 Fall Final Exam**	Dept. or School		Year		proctor	
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※ Your answer must be provided with descriptions how to get the answer.

1. (4 points) Find the local maximum and minimum values and saddle points of $f(x,y) = x^3 - 12xy + 8y^3$.

2. (4 points) Use Lagrange multipliers to find the extreme values of $f(x,y,z) = 3x - y - 3z$ subject to both constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

3. (4 points) Calculate the double integral

$$\iint_R x\sin(x+y)\,dA, \qquad R = [0, \frac{\pi}{3}] \times [0, \frac{\pi}{6}]$$

4. (3 points) Evaluate the integral $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{y^3+1}\, dy\, dx$.

5. (3 points) Find the surface area of the part of the sphere $x^2+y^2+z^2=4z$ that lies inside the paraboloid $z=x^2+y^2$.

6. (3 points) Find the volume of the region E that lies between the paraboloid $z=24-x^2-y^2$ and the cone $z=2\sqrt{x^2+y^2}$.

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7. (3 points) Evaluate the triple integral $\iiint_E (x^2+y^2)\,dV$ where E is the solid that lies above the cone $\phi = \dfrac{\pi}{3}$ and below the sphere $\rho = 2$.

8. (4 points) Evaluate the integral $\iint_R xy\,dA$, where R is the region in the first quadrant bounded by the lines $y=x$ and $y=3x$ and the hyperbolas $xy=1, xy=3$ by using the transformation $x=\dfrac{u}{v}, y=v$.

9. (4 points) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = xy\mathbf{i} + (x-y)\mathbf{j}$ and C consists of the parabola $y=-x^2$ from $x=-2$ to $x=2$ followed by the line segment from $(2,-4)$ to $(5,1)$.

10. (4 points) If $\mathrm{F}(x, y) = (1 - ye^{-x})\mathrm{i} + e^{-x}\mathrm{j}$, evaluate the line integral $\int_C \mathrm{F} \cdot d\mathrm{r}$ where C is the curve given by $y = \sin\left(\frac{\pi}{2}x\right) + 1,\ 0 \le x \le 1$.

11. (4 points) Using Green's Theorem, evaluate $\int_C \frac{y}{x+1} dx + 2xy\, dy$, where C is the positively oriented curve consisting of the curve $y = x^2$ from $(0,0)$ to $(1,1)$ and the line form $(1,1)$ to $(0,0)$.